

# Application of Digital Sliding Modes to Synchronization of the Work of Two Pneumatic Rodless Cylinders

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## Abstract

The paper considers the problem of ensuring synchronized movement of two pneumatic rodless cylinders. This problem is very often for some machines. The control system for synchronization was designed by applying the theory of control system with variable structure. The algorithm of control is based on digital sliding mode. The goal of synthesis of control is to achieve movement of the system in the space of state on a pre-given hyper-surface, in systems of the higher order, i.e. on a line (most frequently a straight line), in systems of the second order. To do so, what must be ensured is the transfer of the system's state from any initial state to the given hyper-surface and its subsequent movement on it in the sliding regime. Measuring coordinates of state (positions and velocities) directly on the rodless cylinders is supposed to be possible. It is shown that such a system ensures quick synchronization of rodless cylinders under different initial conditions (loads and/or positions). The applied algorithm was compared with conventional algorithms of control. The quality of work of the considered system was illustrated by computer simulation.

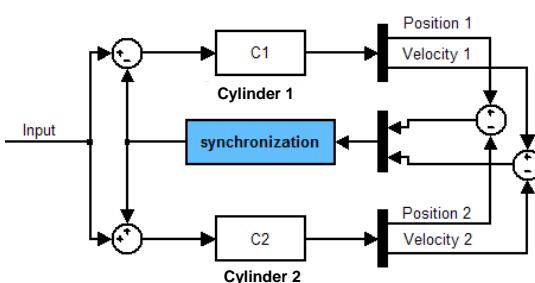
**Key words:** pneumatic rodless cylinder, variable structure system, digital sliding mode, synchronization

## 1. INTRODUCTION

Pneumatic rodless cylinders are widely applied in flexible automation of machines and manufacturing processes. As with other types of actuators (e.g. electric motors), it is frequently demanded that two or more pneumatic rodless cylinders move synchronously. Since these rodless cylinders can have different initial conditions and different loads, it is necessary, as soon as possible, to achieve synchronization of further movement to a given position value. This problem was examined in detail by Unbehauen and Vakilzadeh [1,2,3,4] for various types of actuators, with the application of various algorithms of control of conventional type (P, PI, PID). The basic idea is represented in Fig. 1.

The referent signal is supplied to the system's input. Control signals of rodless cylinders C1 and C2, respectively, are obtained as the sum and subtract of the referent input and the signal of the feedback, generated by the control mechanism (synchronizer). The feedback's signal is generated as the function of the difference of vectors of state (of position and velocity) for rodless cylinders C1 and C2. It is also

possible to view the system as a control system with an observer, where, apart from the effect of the signal of the observation error on the observer's input, the effect on the object's input is also achieved. In the given case, C2 is the 'observer', while C1 is the 'object'. The basic difference from the system with the observer lies in the fact that here both the 'observer' and the 'object' are dynamic elements with the same dynamic properties and approximately the same parameters.



**Figure 1.** Schematic representation of synchronization system

The main goal of this paper is exploration of the possibility of applying the algorithm of control of the

variable structure with sliding working regime to the problem of synchronization. The basic features of sliding regimes, known to a small circle of experts in the field of automatic control, are:

- theoretical invariance to the external load and internal perturbations (parameters uncertainty) if machining conditions are satisfied [5] and practical robustness;
- the character of the system's movement is known in advance;
- the movement does not depend on the object's parameters and control, but only on control parameters;
- what is necessary is not exact knowledge of the object's parameters, but only of the range of their possible change;
- it is easier to ensure the system's stability by decomposing the problem of stability into two simpler sub-problems;
- lowering the order of the differential equation which describes movement.

The paper is organized in the following way: the second part explains the method of obtaining mathematical models of control objects. The third part presents an outline of the control algorithm on the basis of [6,7,8,9,10,11]. The fourth part presents the effects of

the application of the above-mentioned algorithm of control in comparison with conventional algorithms [1,2,3,4] by means of computer simulation.

## 2. MATHEMATICAL MODEL OF THE SYSTEM

A common positional pneumatic servo system with two rodless cylinders is presented in Fig. 2. It is composed of two double-acting pneumatic rodless cylinder, designated as 1.0 and 2.0 and proportional valve 5/3 designated as 1.1 and 2.1 for each rodless cylinder.

There are  $F_1$  and  $F_2$  for external load,  $P_{i1}$  and  $P_{i2}$ ,  $i=1, 2$  are the absolute pressures in the each rodless cylinder's chambers,  $Ps$  is the absolute pressure supply and  $u_1$  and  $u_2$  and  $u_n$  are the input DC current and voltage of proportional valve produced by synchronization block.

The rodless cylinder ports are connected to a proportional valve, and cylinder piston motion is obtained by modulating the compressed air flow into and out of the rodless cylinder chambers.

A proportional valve provides this modulation [6,7,8], by adequate control algorithm. The digital sliding mode control is used in this paper and synchronization system from Fig.1.

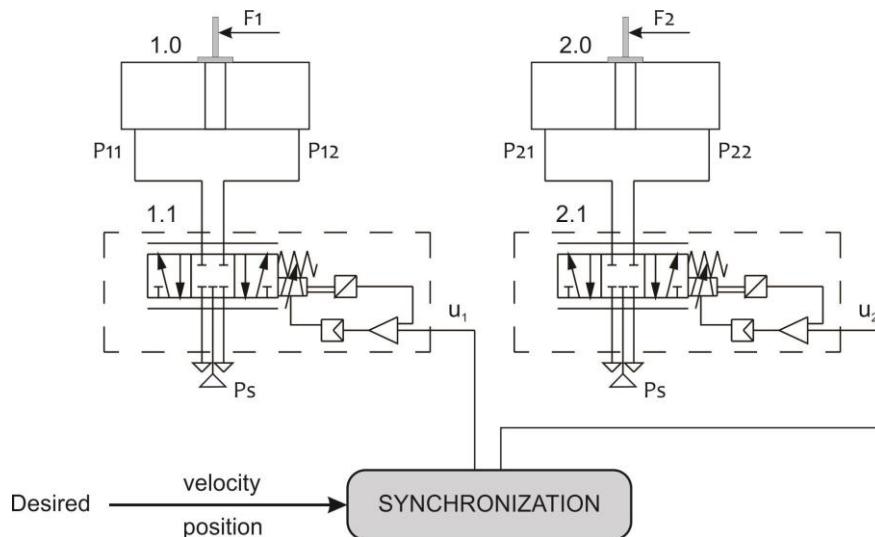


Figure 2. Pneumatic scheme of the servo system for synchronization

The second order linearized transfer function of the presented system [6,7,8], can be represented as:

$$G(s) = \frac{K_p}{s^2 + K_1 s + K_2} \quad (1)$$

where  $K_p$ ,  $K_1$  and  $K_2$  are coefficients of transfer function and they depend of:

- rodless cylinder dimension,
- tubing, proportional control valve,
- compressed air characteristics and other parameters.

All parameters are detail explained in papers [6,7,8].

## 3. DIGITAL SLIDING MODE CONTROL ALGORITHM

The algorithm of control, whose application to the problem of synchronizing pneumatic actuators is examined in this paper, belongs to the group of digital algorithms of control of variable structure. The goal of synthesis of control is to achieve movement of the system in the space of state on a pre-given hyper-surface, in systems of the higher order, i.e. on a line (most frequently a straight line), in systems of the second order. To do so, what must be ensured is the transfer of the system's state from any initial state to the given hyper-surface and its subsequent movement on it in the sliding regime. This means that the system's

phase trajectories all go into the given hyper-surface. Since it is selected so that it passes through the outcome of the space of state, which represents the state of equilibrium, asymptotic stability of the system is also ensured. In this way, the system is brought into equilibrium according to a pre-given trajectory, which may also have attributes of optimality. To summarize, the movement of these system has three phases: (I) the phase of reaching the hyper-surface; (II) the phase of the sliding regime; (III) the phase of steady state.

If the sliding hyper-surface is marked as  $s(x)$ , the conditions are met by satisfying the inequality

$$s(\mathbf{x})\dot{s}(\mathbf{x}) < 0 \quad (2)$$

This condition can be satisfied by applying various algorithms of control. However, they must contain a relay component of type, which may give rise to parasitic movements (chattering) in the area of the hyper-surface  $s(x)=0$  even in the steady state. Such movements are especially intrusive in electromechanical systems

$$U_0 \operatorname{sgn}\{s(\mathbf{x})\}, U_0 > 0 \quad (3)$$

The algorithm applied in this paper eliminates or minimizes the problem of vibration to a tolerable level. The control is formed so that it has two components: a relay component, which ensures safe and quick transfer of the system's state near the sliding hyper-surface without intersecting it, and a linear component, which brings the system's state into  $s(x)=0$  in the following step (during one discretization period).

Since the control will be digitally implemented, it is necessary to perform time-discretization of the model (1). If  $T_d$  denotes the sampling period and

$$\mathbf{A}_\delta(T_d) = \frac{e^{AT_d} - \mathbf{I}_n}{T_d}, \quad \mathbf{b}_\delta(T_d) = \frac{1}{T_d} \int_0^{T_d} e^{A\tau} \mathbf{b} d\tau \quad (4)$$

the discrete-time state-space model of the nominal system, by using  $\delta$ -transform, can be expressed in the form:

$$\delta \mathbf{x}(kT_d) = \mathbf{A}_\delta(T_d) \mathbf{x}(kT_d) + \mathbf{b}_\delta(T_d) \mathbf{u}(kT_d) \quad (5)$$

The commonly used linear scalar switching function  $s(x)$  in sliding mode control (SMC) systems that defines a sliding hyperplane in the state space ( $s=0$ ), along which the sliding mode is organized, is chosen as

$$s = \mathbf{c}_\delta(T_d) \mathbf{x} \quad (6)$$

where  $\mathbf{c}_\delta(T_d)$  is the switching function vector of appropriate dimension.

The detailed control design procedure of the selected discrete-time SMC (DSMC) algorithm is given in [9,10]. Here, only the important relations will be briefly recounted. The positioning control law is obtained as:

$$u_c = -\mathbf{c}_\delta(T_d) \mathbf{A}_\delta(T_d) \mathbf{x}(k) - \Phi(s(k), \mathbf{X}(k)) \quad (7)$$

The switching function vector  $\mathbf{c}_\delta(T_d)$ , which exclusively defines system dynamics in the sliding mode, is selected according to the relation:

$$\mathbf{c}_\delta(T_d) = [\mathbf{c}_1(T_d) | 1] \mathbf{P}_1^{-1}(T_d) \quad (8)$$

The values required for calculating matrix  $\mathbf{c}_\delta(T_d)$ , are obtained through the following equations [10]:

$$c_i(T_d) = \frac{1}{(i-1)!} \frac{d^{i-1} \prod_{j=1}^{n-1} (\delta - \delta_j(T_d))}{d\delta^{i-1}} \Big|_{\delta=0} \quad (9)$$

$$\delta_i(T_d) = \frac{e^{-\alpha_i T_d} - 1}{T_d}, \quad \alpha_i > 0, \quad i \neq j \Rightarrow \alpha_i \neq \alpha_j, \quad i, j = 1, \dots, n-1 \quad (10)$$

$$\begin{aligned} \mathbf{P}_1^{-1}(T_d) &= [b_\delta(T_d) \dots A_\delta^{n-1}(T_d) b_\delta(T_d)] A_d \\ A_d &= \begin{bmatrix} a_1(T_d) & \dots & a_{n-1}(T_d) & 1 \\ a_2(T_d) & \dots & 1 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 1 & \dots & 0 & 0 \end{bmatrix} \end{aligned} \quad (11)$$

Function  $\Phi(s, X)$  is defined as:

$$\Phi(s, X) = \Phi(s) = \min\left(\frac{|s|}{T_d}, \sigma + \rho |s|\right) \operatorname{sgn}(s); \quad 0 \leq \rho T_d < 1, \quad \sigma > 0 \quad (12)$$

Parameters  $\sigma$  and  $\rho$  define sliding mode reaching dynamics and should be chosen to provide as short as possible reaching phase.

#### 4. SYNCHRONIZATION OF THE SYSTEM WITH TWO PNEUMATIC RODLESS CYLINDERS

As was said in the introduction, synchronization of the work of two pneumatic rodless cylinders is a sizeable problem, which can be solved by using adequate control.

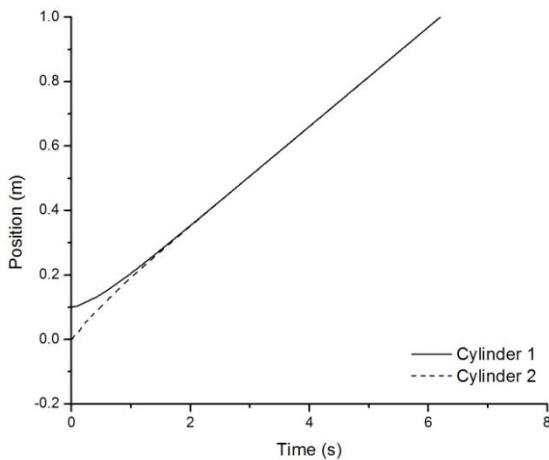
The pneumatic system to be considered in this section consists of two pneumatic double-acting rodless cylinders whose work should be synchronized, since they have different initial positions. The system's model, given in state space form by equation (13),

$$G(s) = \frac{4.065}{s^2 + 180s} \quad (13)$$

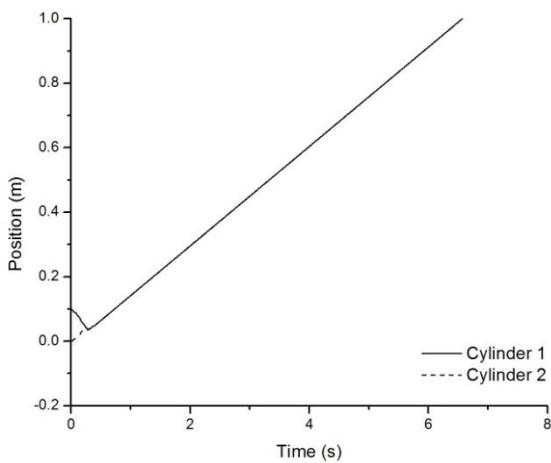
The sliding hyper-surface coefficient is calculated by means of equation (8), for  $\alpha=15s-1$ . Parameters  $q$  and  $\sigma$  are taken to be 0 and 24, respectively. Vector  $\mathbf{c}_\delta(T) = [-16.428 \ -2.035558]$  and  $\mathbf{c}_\delta(T) \mathbf{A}_\delta(T) = [0 \ 87.8456]$ . Different initial positions are reflected in the fact that the cylinder piston of the first rodless cylinder is already somewhat drawn out, whereas the cylinder piston of the other rodless cylinder is completely pulled in.

The problem of synchronization is to bring, within a short period of time, the positions of both rodless cylinders ( $x_1$  and  $x_2$ ) into equal positions, with no differences in further work. In order to successfully solve this problem of synchronization in the manner described in section 3, the algorithm of digital control, given in equation (7), is used, which ensures that errors in positions and velocities of the cylinder pistons are eradicated within a short period of time.

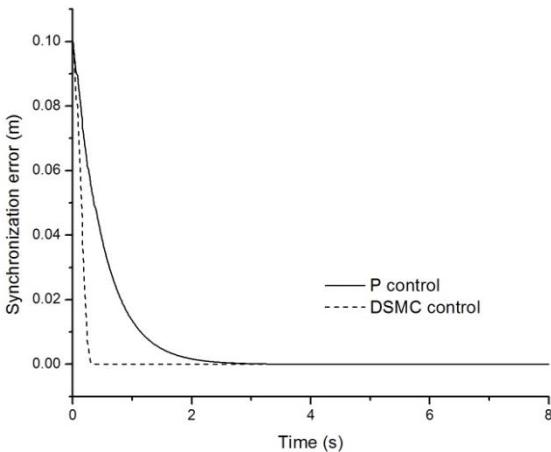
To confirm that the algorithm of control, which enables quick synchronization, was adequately selected, Fig. 3 and Fig 4, shows simulated results of the positions of standard pneumatic rodless cylinders.



**Figure 3.** Simulated results of synchronization of the standard pneumatic rodless cylinders with P controller



**Figure 4.** Simulated results of synchronization of the standard pneumatic rodless cylinders with DSMC



**Figure 5.** Simulated results of the position differences between two standard pneumatic rodless cylinders, when P controller and DSMC are used

As has been mentioned, initial positions of the rodless cylinders differ; in this case, the cylinder piston of the first rodless cylinder C1 is at initial position  $x_1=0.1\text{m}$ , whereas the cylinder piston of the other rodless cylinder C2 is completely pulled in, i.e.  $x_2=0\text{ m}$ . Fig. 3 shows the

work of the system when synchronization is performed with P controller, while Fig. 4 shows the result of the synchronization when DSMC is applied.

Fig. 5 shows the diagram of position differences between two pneumatic rodless cylinders, when P controller and DSMC are used.

## 5. CONCLUSION

The paper outlines the problem of synchronizing the work of two rodless cylinders, and the algorithm of digital control which solves this problem is given. The algorithm of control serves its purpose, since it ensures synchronization of the work of the rodless cylinders within a short period of time, which was demonstrated and explained in section 4. This way of synchronizing double-acting rodless cylinders simplifies and reduces the price of realization to a significant extent.

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